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Original Article

The Generalized Parallelism Theorem for Real and Vector-Valued Functions

A Geometric and Functional Approach to Non-Intersection Conditions

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KEYWORDS

Functional parallelism, vertical translation, non-intersection theorem, real-valued functions,vector-valued functions, graph theory in analysis, normed function spaces

ABSTRACT

This work introduces a new formalization of the concept of functional parallelism between real-valued functions, extending the classical meaning of parallel lines in the plane to generic continuous functions over a common domain. A theorem, referred to as the generalized paralleli-sm theorem, is proposed to characterize the conditions under which two functions can be considered parallel in an expanded geometric sense. The rigorous definition is based on vertical translations and demonstra-tes that such a relationship implies the non-intersection of the respecti-ve graphs. This perspective unifies and formalizes an idea often implicit in the treatment of geometric transformations, offering an elegant and mathematically consistent generalization of traditional parallelism (1, 2).

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1. Introduction

In the context of analytic geometry, two lines in the cartesian plane are said to be parallel if they never intersect, a condition that is equivalent to having the same slope, as defined by the criterion of parallelism. This concept is fundamental in the study of geometric figures and linear transformations (3, 4). We now extend this concept to real-valued functions in a way that preserves the geometric intuition, namely, that the graphs of the functions never intersect, and that one function is a vertical translation of the other.

2. Definition of functional parallelism

Let f(x) and g(x): D → R be two real-valued functions defined on a common domain $D \subseteq R$. We say that: the functions f(x) and g(x) are parallel if there exists a real constant $k \neq 0$ such that:

$$g(x) = f(x) + k \qquad \forall x \in R$$

This relation implies that, for every point in the domain, the value of g(x) differs from that of f(x) by a constant amount. Geometrically, the graph of g(x) is a vertical translation of the graph of f(x) by k units.

3. Generalized parallelism theorem

Let f(x) and g(x): D → R be two functions such that g(x) = f (x) + k, with $k \in R \setminus \{0\}$. Then:

$$f(x) \neq g(x) \qquad \forall x \in R$$

That is, the graphs of f(x) and g(x) do not intersect (4, 5).

Suppose, for the sake of contradiction, that there exists a point $x_0 \in D$ such that $f(x_0) = g(x_0)$. But by hypothesis:

$$f(x_0) = g(x_0) + k \Rightarrow f(x_0) = f(x_0) + k \Rightarrow k = 0$$

which contradicts the assumption that $k \neq 0$.

Therefore, no point of intersection can exist (1, 6). For example: consider the functions f(x) = sin(x) and its vertical translation g(x) = sin(x) + 1,5. As shown in the corresponding graph, the two functions remain at a constant vertical distance across the entire domain and never intersect. This provides a visual confirmation of the definition of parallel functions introduced earlier (Figure1) (4).

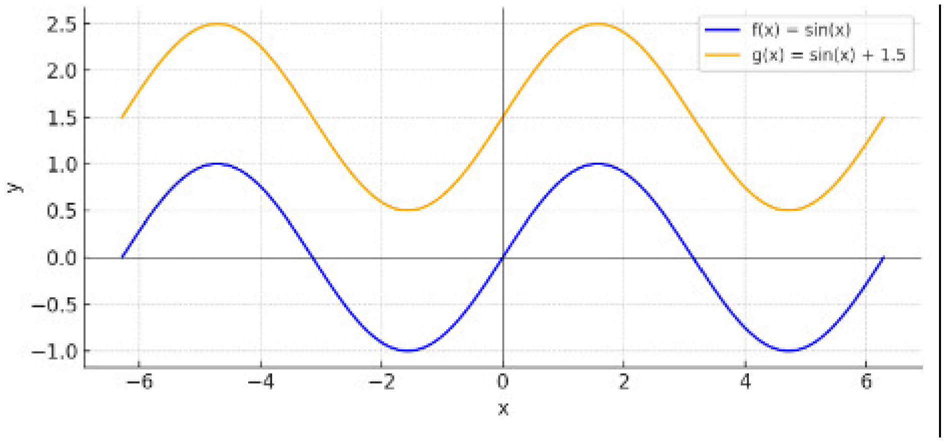

Figure 1. *Comparison between the graph of a function f(x) = sin(x) and its vertical translation g(x) = sin(x) + 1,5, illustrating the generalized parallelism. As shown, the two graphs maintain a constant vertical distance throughout the domain and never intersect, visually confirming the definition and properties of functionally parallel curves.*

4. Extension to vector-valued functions

The concept of parallelism between functions can also be extended to vector-valued functions and curves in space. Consider a vector-valued function r(t): $R \to R^3$ defined as:

$$r(t) = (x(t); y(t); z(t))$$

We can define a vector-valued function parallel to r(t) as:

$$r_2(t) = r(t) + \vec{k}$$

where $\vec{k}$ is a constant vector in R^3. In this case, the graph of $r_2(t)$ is a curve in space perfectly parallel to r(t), rigidly translated along the direction of $\vec{k}$. A concrete example is the helical curve:

$$r(t) = (\cos(t), \sin(t), t)$$

and its parallel curve:

$$r_2(t) = (\cos t, \sin t, t + 2)$$

The two graphs do not intersect and remain equidistant along the z axis (Figure 2) (2).

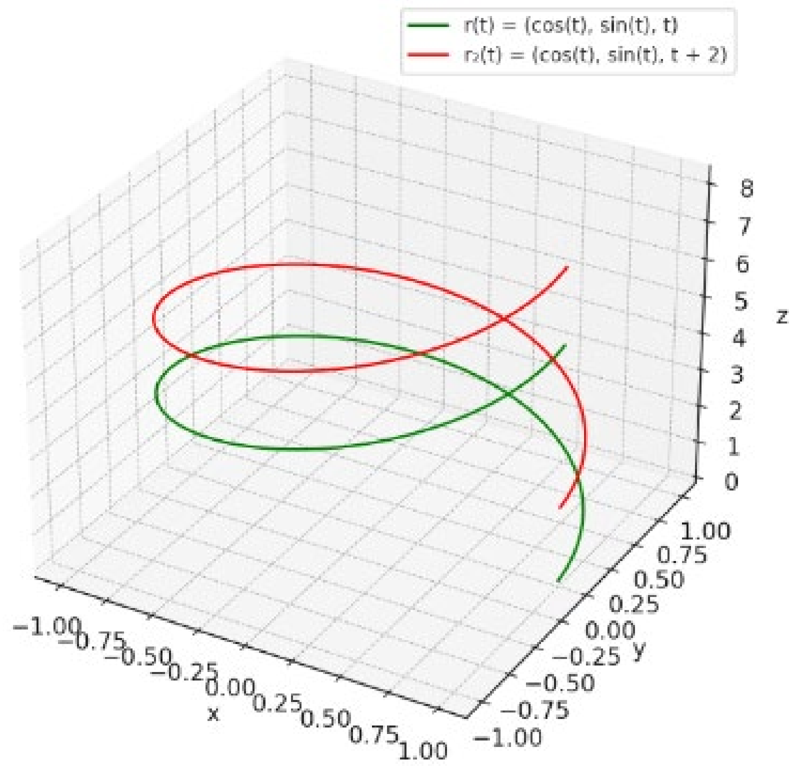

Figure 2. *Comparison between the graph of a three-dimensional function r(t) = (cos(t), sin(t), t) and its vertical translation $r_2(t)$ = (cos t, sin t, t + 2) along the z axis, illustrating the generalized parallelism. As shown, the two curves remain at a constant distance in the z direction and do not intersect, visually confirming the extension of the concept of functional parallelism to curves in space.*

5. Extension to non-canonical context

In this section, we explore the applicability of the Generalized Parallelism Theorem beyond the canonical cases of continuous functions defined on connected domains. The following extensions fall outside the scope of the classical theorem but represent interesting variations that deserve theoretical attention.

Discontinous functions

If the functions f(x) and g(x) are not continuous but still satisfy the pointwise relation

$$g(x) = f(x) + k$$

for every x in the common domain D, then we can speak of *analytic parallelism*. However, the geometric concept of parallelism between the graphs loses its usual meaning, since it is no longer possible to define a continuous constant distance between the two curves (6). In this case, we can still say that f(x) and g(x) are *pointwise parallel*, in the sense that they maintain a constant offset wherever both functions are defined.

Non-connected domains

When the domain D consists of multiple disjoint connected components, for example, a union of separate intervals, the definition can be extended by introducing the notion of *local parallelism*. Two functions f(x) and g(x) are *locally parallel* if, on each connected component $C_i \subseteq D$, there exists a constant

k_i such that

$$g(x) = f(x) + k_i \qquad \forall x \in C_i$$

If all the constants k_i coincide, then f(x) and g(x) are *globally parallel* even on a disconnected domain D. Otherwise, we talk about a family of *piecewise parallel functions* defined on subdomains.

6. Discussion and extension

This definition of functional parallelism can be useful in various contexts:

- Analysis of families of functions of the form g(x) = f (x) + k (5),
- Study of vertical transformations,
- Geometric approach to concepts such as equidistance between graphs,
- Extension to vector-valued functions and curves in R^∞.

A natural extension could be to define a constant functional distance between two graphs, even in multidimensional spaces, by using norms such as the L^∞ norm or the L^2 norm.

7. Conclusion

This work has developed a rigorous generalization of the concept of parallelism, extending it from lines to real and vector-valued functions through a definition based on vertical translations and affine transformations. The Generalized Parallelism Theorem not only characterizes the conditions for non-intersection between graphs but also extends to more complex contexts such as discontinuous functions and non-connected domains, introducing the notions of analytic and local parallelism. This formalization finds application both in educational settings, as a tool to visualize geometric transformations, and in advanced research contexts, particularly in the study of normed spaces and differential geometry (1, 4, 5). Future perspectives of this theory include natural extensions to multivariate functions and to non-Euclidean functional spaces, which may reveal new connections between analysis and geometry. The unifying approach adopted in this work demonstrates how a classical idea can be revitalized through rigorous treatment, simultaneously offering geometric insights and analytical tools for future theoretical developments.

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